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# Vector mesons in $q\bar{q}q\bar{q}$ systems

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## abstract

We discuss the vector mesons, $I^G(J^{PC}) = 1^+(1^{--})$ and $1^-(1^{-+})$ in $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations, decaying into two ground-state octets. We derive a powerful selection rule, valid in the limit of flavor $SU(3)$ symmetry. The octets considered are $\{\pi\}$, $\{\rho\}$, $\{b_1\}$, $\{a_1\}$ and $\{a_2\}$, labeled by the isovector state in the representation.

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An exotic meson, the $\pi_1(1400)$ with $I^G(J^{PC}) = 1^-(1^{-+})$, has been seen to decay into a $p$-wave $\eta\pi$ system [1]. It has been shown [2] that, if the $\eta$ meson is assumed to be a (pure) member of the pion octet and in the limit of flavor $SU(3)$ conservation in its decay, a $p$-wave $\eta\pi$ system belongs to a flavor $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation by the requirement of Bose symmetrization. This implies that the $\pi_1(1400)$ must belong to a family of four-quark states $(q\bar{q} + q\bar{q})$. In contrast, a second exotic meson, the $\pi_1(1600)$ with $I^G(J^{PC}) = 1^-(1^{-+})$, is reported to have a substantial decay mode into $\eta'\pi$ [3]. Whatever its constituent quark content may be—since the $\eta'$ is mostly an $SU(3)$ singlet—it is a member of an octet.

The multiplet $\mathbf{10} \oplus \overline{\mathbf{10}}$ is given in a pictorial form in Fig.1.

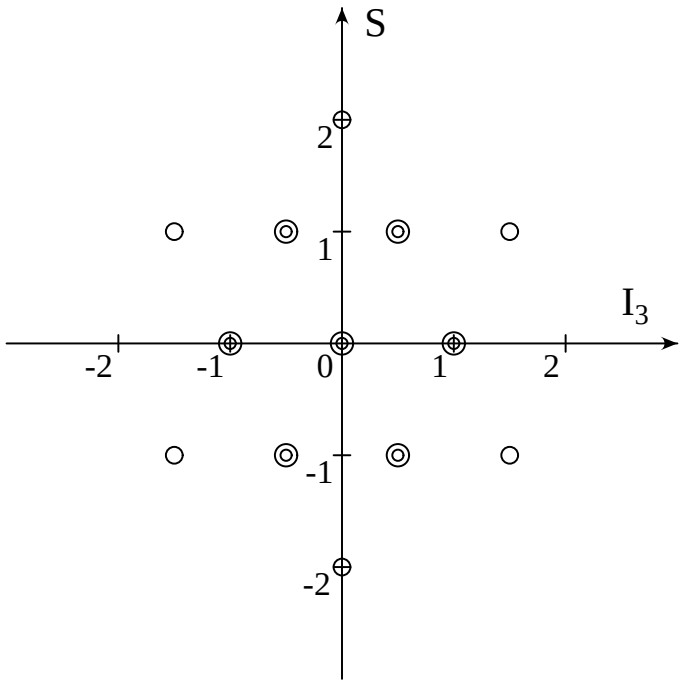


Figure 1: Weight Diagram for multiplet $\mathbf{10}\oplus\overline{\mathbf{10}}$; single circles have just one member of the multiplet, while the double circles indicate two occupancies by the members of the multiplet.

The purpose of this letter is explore the consequence of the existence of other vector mesons predicted to exist in the multiplet $\mathbf{10} \oplus \overline{\mathbf{10}}$ [2]. In the limit of flavor $SU(3)$ symmetry, *the mass of the* $I^G(J^{PC}) = 1^+(1^{--})$ *member must be equal to that of the* $\pi_1(1400)$. We shall adopt the notation $\rho_x(1400)$ for this state. We know that *both* must belong to the $q\bar{q}q\bar{q}$ family of mesons.

A full account of the $J^{PC} = 1^{--}$ and $1^{-+}$ 'vector' mesons with $q\bar{q}q\bar{q}$ entails existence of *two* sets of 'supermultiplets' with 81 members[a] in each. Each supermultiplet has the

structure

$$\mathbb{V} = \{q\bar{q}q\bar{q}\} = 2\times\mathbf{1}\ \oplus\ 4\times\mathbf{8}\ \oplus\ \mathbf{10}\ \oplus\ \overline{\mathbf{10}}\ \oplus\ \mathbf{27} \tag{1}$$

where we consider here three flavors $q = \{u, d, s\}$. Of course, not all members of the multiplets need to support quasi-bound-state systems—with reasonably finite widths—which can be identified experimentally.

Let $\chi^0$ be the wave function for a nonstrange, neutral state in any of the representations listed above (there is at least one such state in each). We now define the supermultiplets through the charge-conjugation operator $\mathbb{C}$

$$\begin{aligned}\mathbb{V}_\zeta: \quad & \mathbb{C}\,|\chi^0(\boldsymbol{n})\rangle = \zeta\,|\chi^0(\boldsymbol{n})\rangle, \qquad \boldsymbol{n} = \mathbf{1},\ \mathbf{8},\ \mathbf{27} \\ & \mathbb{C}\,|\chi^0(\mathbf{10})\rangle = \zeta\,|\chi^0(\overline{\mathbf{10}})\rangle, \quad \mathbb{C}\,|\chi^0(\overline{\mathbf{10}})\rangle = \zeta\,|\chi^0(\mathbf{10})\rangle\end{aligned} \tag{2}$$

where $\zeta = \pm 1$. Thus, $\zeta$ is simply the $\mathbb{C}$ eigenvalue of a $\chi^0$ which belongs to any of the 'self-conjugate' representations **1**, **8** and **27**. The subscript therefore refers to the $C$-parity of the dominant $J^{PC}$ in the family ($\zeta = C$). Each $\mathbb{V}_\pm$ supermultiplet comes with 61 $J^{PC} = 1^{-\pm}$ members belonging to the self-conjugate representations of $SU(3)$. The $\mathbf{10}\oplus\overline{\mathbf{10}}$ members are broken up into 14 $J^P = 1^-$ strange members and *two* sets of three $I^G(J^{PC}) = 1^\pm(1^{-\mp})$ members. Thus the physical states in eigenstates of $\mathbb{C}$ are

$$\chi^0_\pm = \frac{1}{\sqrt{2}}\left[\chi^0(\mathbf{10}) \pm \zeta\,\chi^0(\overline{\mathbf{10}})\right], \quad \mathbb{C}|\chi^0_\pm\rangle = \pm|\chi^0_\pm\rangle \tag{3}$$

The 'quantum number' $\zeta$ has *nothing* to do with the $C$-parity of the nonstrange members of $\mathbf{10}\oplus\overline{\mathbf{10}}$. We assume that one supermultiplet, $\mathbb{V}_+$ or $\mathbb{V}_-$, gives rise to $\pi_1(1400)$ and $\rho_x(1400)$; we shall designate their counterparts by $\pi_1'(1400?)$ and $\rho_x'(1400?)$ in the other supermultiplet. All four states are isospin triplets and belong to $\mathbf{10}\oplus\overline{\mathbf{10}}$ representations.

Let $X$ stand for a nonstrange state belonging to a $\mathbf{10}\oplus\overline{\mathbf{10}}$ representation. The main purpose of this paper is to point out that the $\zeta$'s, as defined through (2), lead to a powerful selection rule regarding the decay $X \to a + b$ where $a$ and $b$ are members of the octets (not necessarily the same). For the purpose, consider the decay modes of a $J^P = 1^-$ nonstrange meson $X$ as outlined below:

$$X \to [\{\pi\}\{\pi\}]_P\,,\ [\{\rho\}\{\rho\}]_{P,F}\,,\ [\{a_1\}\{\pi\}]_{S,D}\,,\ [\{a_2\}\{\pi\}]_D\,, \tag{4a}$$

[a] The number of states in a supermultiplet can derived in two different ways: $\{q\bar{q}+q\bar{q}\} = (\mathbf{1}\oplus\mathbf{8})\otimes(\mathbf{1}\oplus\mathbf{8})$ or $\{qq+\bar{q}\bar{q}\} = (\overline{\mathbf{3}}\oplus\mathbf{6})\otimes(\mathbf{3}\oplus\overline{\mathbf{6}})$.

$$\rightarrow [\{\rho\}\{\pi\}]_P\,, [\{b_1\}\{\pi\}]_{S,\,D} \tag{4b}$$

where the bracket { } refers to the totality of an octet. For example, $\{\pi\}$ stands for $\pi$, $K$, $\bar{K}$ and $[\eta]_8$. (we use $[\ ]_8$ to denote the octet component of a particle; in addition, we follow a common practice of writing $[a\,b]_L$ to indicate an orbital angular momentum $L$ between $a$ and $b$.) The $SU(3)$ wave functions $\phi$ (real by definition) for nonstrange neutral members of $\{a\}\,\{b\}[4]$ are transformed,[b] under $\mathbb{C}$,

$$\mathbb{C}\,\phi(\overline{\mathbf{10}}) = g\,\phi(\mathbf{10}), \quad \mathbb{C}\,\phi(\mathbf{10}) = g\,\phi(\overline{\mathbf{10}}) \tag{5}$$

where $g = C_a \cdot C_b = \pm 1$. $C$ refers to the $C$-parity of the octets involved in the final state. Hence $g$ is equal to a product of two $G$-parities for the *isovector* members of $\{a\}$ and $\{b\}$. For example, $g = +1$ for the decay $\{\pi\}\{\pi\}$ and likewise for all the final states listed in $(4a)$, while $g = -1$ for $(4b)$.

Because of $C$-parity conservation, the amplitudes for the decay $X \rightarrow a + b$ must obey the relationship

$$\langle\phi(\overline{\mathbf{10}})|\mathcal{M}|\chi^0(\overline{\mathbf{10}})\rangle = \kappa\,\langle\phi(\mathbf{10})|\mathcal{M}|\chi^0(\mathbf{10})\rangle \tag{6}$$

where $\kappa = \pm 1$. (The $C$-parity conservation requires that the squares of the amplitudes be the same.) We now insert $\mathbb{C}^\dagger\,\mathbb{C} = I$ next to the operator $\mathcal{M}$ on the left side of (6) and use (2) and (5)—to obtain $\zeta = \kappa\,g$. *This is our main result.* It can be stated as follows: the phase $\zeta$, which determines to which supermultiplet $X$ belongs, specifies how it should couple to a decay product consisting of two octets. Our selection rule can be succinctly stated

$$\begin{aligned} X &\rightarrow [\{a\}\,\{b\}]_+\,, \qquad X \not\rightarrow [\{a\}\,\{b\}]_- \\ X' &\rightarrow [\{a\}\,\{b\}]_-\,, \qquad X' \not\rightarrow [\{a\}\,\{b\}]_+ \end{aligned} \tag{7}$$

where $X \in \mathbb{V}_+$ and $X' \in \mathbb{V}_-$ if $\kappa = +1$ and *vice versa* if $\kappa = -1$. The subscript to $\{a\}\,\{b\}$ stand for $g = C_a \cdot C_b$. In another words, the allowed decay modes for $X \rightarrow a + b$ must obey the rule $\zeta \cdot C_a \cdot C_b = +1$ in the limit of $SU(3)$ symmetry, if $\kappa = +1$. But a state $X$ *cannot* come with both $\kappa = +1$ and $\kappa = -1$ at the same time. So the selection rule (7) should be true always, except that there is no *a priori* way of knowing to which supermultiplet the $X$ belongs.

[b] There is an overall phase $\eta$ in the defining formula (8.2) of Ref.[4], which has been set to $+1$. However, this is really not necessary for our purposes, as it can be absorbed into $\kappa$ of (6).

Let $\chi$ be the wave function for the isovector meson $X$ in *any* charge state. Because $X$ and $X'$ are isovectors, the $\chi$'s transform under the $G$-parity operator

$$\begin{aligned} &\mathbb{G}\,|\chi(\mathbf{10})\rangle = -\zeta\,|\chi(\overline{\mathbf{10}})\rangle, \quad \mathbb{G}\,|\chi(\overline{\mathbf{10}})\rangle = -\zeta\,|\chi(\mathbf{10})\rangle, \\ &\chi_\pm = \frac{1}{\sqrt{2}}\,\left[\chi(\mathbf{10}) \mp \zeta\,\chi(\overline{\mathbf{10}})\right], \quad \mathbb{G}|\chi_\pm\rangle = \pm|\chi_\pm\rangle \end{aligned} \tag{8}$$

Four $G$-parity eigenstates, together with their allowed decay modes, are summarized in Tables Ia and Ib (for $\kappa = +1$).

Table Ia: Decay Modes[a] for $X\,(\zeta = +1) \to [\{a\}\,\{b\}]_+$

| $I^G(J^{PC})$ | $\sqrt{2}\,\chi_\pm$ | $\{\pi\}\{\pi\}$ [b] | $\{\rho\}\{\rho\}$ [b] | $\{a_1\}\{\pi\}$ [c] | $\{a_2\}\{\pi\}$ |
|---|---|---|---|---|---|
| $1^+(1^{--})$ | $\chi(\mathbf{10}) - \chi(\overline{\mathbf{10}})$ | $\pi\,\pi,\ K\bar{K}$ | $\rho\rho,\ K^*\bar{K}^*$ | $a_1\,\pi,\ K_{1A}\bar{K}$ | $a_2\,\pi,\ K_2^*\bar{K}$ |
| $1^-(1^{-+})$ | $\chi(\mathbf{10}) + \chi(\overline{\mathbf{10}})$ | $\pi[\eta]_8$ | $\rho[\omega]_8$ | $[f_1]_8\,\pi$ | $[f_2]_8\,\pi$ |

Table Ib: Decay Modes[a] for $X'\,(\zeta = -1) \to [\{a\}\,\{b\}]_-$

| $I^G(J^{PC})$ | $\sqrt{2}\,\chi'_\pm$ | $\{\rho\}\{\pi\}$ [b] | $\{b_1\}\{\pi\}$ [b c] |
|---|---|---|---|
| $1^+(1^{--})$ | $\chi'(\mathbf{10}) + \chi'(\overline{\mathbf{10}})$ | $[\omega]_8\,\pi$ | $[h_1]_8\,\pi$ |
| $1^-(1^{-+})$ | $\chi'(\mathbf{10}) - \chi'(\overline{\mathbf{10}})$ | $\rho\,\pi,\ K^*\bar{K}$ | $b_1\,\pi,\ K_{1B}\bar{K}$ |

[a] When both $n\bar{n}$ ($n = \{u, d\}$) and $s\bar{s}$ decays are allowed, the predicted branching ratio is $B(n\bar{n})/B(s\bar{s}) = 1/2$.

[b] The subscripts 8 denote the 'octet' component.

[c] The $K_{1A}$ and $K_{1B}$ are nearly equal mixtures of the $K_1(1270)$ and $K_1(1400)$.

It should be understood that, for each $J^{PC}$, an allowed decay of $X$ (table Ia) is forbidden for $X'$ (table Ib) and *vice versa*.

A further insight can be gained by considering the transformation properties of the wave functions in 'self-conjugate' representations. Such a wave function $\chi^0(\boldsymbol{n})$ for nonstrange, neutral members has already been given in (2), where $\boldsymbol{n} = \mathbf{1}$, $\mathbf{8}$ or $\mathbf{27}$. The $\chi^0(\boldsymbol{n})$'s are $\mathbb{C}$ eigenstates with the eigenvalues $\zeta$. Now let it decay into $\{a\}\,\{b\}$. Then, under $\mathbb{C}$, the final-state wave function $\phi(\boldsymbol{n})$ must transform according to (5), where $\mathbf{10}$ and $\overline{\mathbf{10}}$ are now

replaced by $\boldsymbol{n}$

$$\mathbb{C}\,|\phi(\boldsymbol{n})\rangle = g\,|\phi(\boldsymbol{n})\rangle \tag{9}$$

The eigenvalue is again given by $g = C_a \cdot C_b = \pm 1$. Consider the decay matrix element

$$\langle\phi(\boldsymbol{n})|\mathcal{M}|\chi^0(\boldsymbol{n})\rangle \tag{10}$$

and carry out the same process applied to (6). We then conclude that $\zeta \cdot C_a \cdot C_b = +1$. [c] So an $X$ in a self-conjugate representation $\boldsymbol{n}$ obeys the same selection rule it does in $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations with $\kappa = +1$. The phase $\zeta$ is the $C$-parity of a representation $\boldsymbol{n}$, whereas the $\zeta$ in $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations imposes conditions on the allowed decays of $X$ but *not* its $C$-parity.

In summary, we expect two states of $I^G(J^{PC}) = 1^+(1^{-+})$ mesons, $\pi_1(1400)$ and $\pi_1'(1400?)$, and their vector partners with $I^G(J^{PC}) = 1^-(1^{--})$, $\rho_x(1400)$ and $\rho_x'(1400?)$, all belonging to $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations of flavor $SU(3)$. The $\pi_1(1400)$ and $\rho_x(1400)$ belong in the supermultiplet $\mathbb{V}_\pm$, whereas the $\rho_x'(1400?)$ and $\pi_1'(1400?)$ are members of $\mathbb{V}_\mp$. Since the $\pi_1(1400)$ is seen to decay into $\eta\pi$, its partner $\rho_x(1400)$ should decay into $\pi\pi$ and $K\bar{K}$, with a branching ratio $B(\pi\pi)/B(K\bar{K}) = 1/2$. In addition, the decay $\pi_1(1400) \to \eta\pi$ implies that the same state *cannot* decay into $\rho\pi$ in the limit of $SU(3)$. The Crystal Barrel Collaboration reported[5] observation of a $I^G(J^{PC}) = 1^+(1^{-+})$ state [d] decaying into $\rho\pi$. This could be the $\pi_1'(1400?)$ in the $\mathbf{10} \oplus \overline{\mathbf{10}}$ representation in $\mathbb{V}_\mp$ or a member of one of the $J^{PC} = 1^{-+}$ octets in $\mathbb{V}_\pm$.

Finally, as an illustration of the selection rule, consider production of the $\pi_1(1400)$ in two charge modes

$$\pi^- p \to \pi_1^-(1400)\, p \to \eta\pi^-\, p \tag{11a}$$

$$\pi^- p \to \pi_1^0(1400)\, n \to \eta\pi^0\, n \tag{11b}$$

It has been shown[1] that Reaction (11$a$) is mediated by natural-parity exchange, meaning that the exchanged Reggeons (neutral) should correspond to $\rho^0$, $f_2$ or the Pomeron. We see that Reaction (11$a$) can proceed via $f_2$ only, since the $\pi_1(1400)$ is not supposed to

[c] Note that replacing $\mathbf{10}$ and $\overline{\mathbf{10}}$ by $\boldsymbol{n}$ in (6) leads to a trivial identity with $\kappa = +1$. However, the result $\zeta = g$ for self-conjugate representations relies on the phase convention $\eta = +1$ given in (8.2) of Ref. [4].

[d] The results are consistent with a single state with mass at $\sim 1450$ MeV and its width fixed at 310 MeV; however, there are in fact indications of two states at 1405 and 1650 MeV.

couple to $\rho\pi$ in the limit of flavor $SU(3)$ (see Table Ia) and the Pomeron is an $SU(3)$ flavor singlet. Reaction ($11b$) [6], on the other hand, requires exchange of charged Reggeons. But the charged $\rho$, for production by natural-parity exchange, must be suppressed for the same reason stated above. Moreover, Table Ia shows that the $\pi_1(1400)$ production is not possible via unnatural-parity exchange for Reaction ($11b$). The $\pi_1'(1400?)$ production is possible via natural- and unnatural-parity exchange (i.e. $\rho$ and $b_1$ exchange—see Table Ib); but then it should *not* be decaying to $\eta\pi$ in the limit of $SU(3)$. We see that the selection rule identified in this paper has important experimental consequences. However, a note of caution should be given here; we do not know the mass difference of the vector mesons in $\mathbb{V}\pm$. But, if they are sufficiently close, they will mix, and this may weaken the selection rules.

We have thus identified, for the first time, a powerful selection rule for an exotic meson belonging to $\mathbf{10} \oplus \overline{\mathbf{10}}$ representations. In a follow-up article, we will further discuss experimental ramifications of the four exotic states in these representations.

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# References


[1] D. R. Thompson *et al.*, Phys. Rev. Lett. **79** (1997) 1630; S. U. Chung *et al.*, Phys. Rev. D **60** (1999) 092001; A. Abele *et al.*, Phys. Lett. **B423** (1998) 175; A. Abele *et al.*, Phys. Lett. **B446** (1999) 349.

[2] S. U. Chung, E. Klempt and J. G. Körner, Eur. Phys. J. A **15** (2002) 539.
The reader may consult the references therein for a comprehensive list of previous publications on this topic.

[3] G. M. Beladidze *et al.*, Phys. Lett. **B313** (1993) 276; A. Zaitsev, *Proc. Eighth International Conf. on Hadron Spectroscopy, Beijing, China (1999),* edited by W. G. Li, Y. Z. Huang and B. S. Zou, Nucl. Phys. A675 (2000) 155c; E. Ivanov *et al.*, Phys. Rev. Lett. **86** (2001) 3977.

[4] J. J. de Swart, Rev. Mod. Phys. **35** (1963) 916.

[5] F. Meyer-Wildhagen, Proc. PANIC2002, to be published in Nucl. Phys. A.

[6] S. A. Sadovsky, *Proc. Fifth Biennial Conf. on Low-Energy Anitproton Physics, Villasimius, Italy (1998),* edited by C. Cicalò, A. De Falco, G. Puddu and S. Serci, Nucl. Phys. A655 (1999) 131c.